Sicherheit in Technik und Chemie

BAM Bundesanstalt für Materialforschung und -prüfung

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Measurement- and Model-based Structural Analysis for Early Damage Detection

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Overview

Parts in my dissertation

1. Academic perspective: The relationship between finite element method and least-squares adjustment in point of view of Adjustment Theory

2. Practical Aspect: Early Damage Detection and Localisation in structure by means of finite element method and least-squares adjustment

Overview

Main focus of this presentation is the practical aspect:

According to the motto “too many cooks spoil the broth”, how can we bridge the connection between model and measurement by just coupling the methods of finite element and least-squares?

Closing the gap of model and measurement

Model

Measurement

Integrated Analysis

Modeling

Numeric

Optimization

Measurements

Continuum Mechanics

Adjustment Calculation

Finite Element Method

Experiments

Overview

The four aims in structural health monitoring in civil engineering:
1. the detection
2. the localisation
3. the causes
4. the prognoses of damages

Laying down the fundamental framework especially for number 3

Field Equations from Continuum Mechanics

Elastostatic case

$$0 = C_{ijkl} \frac{\partial^2 u_k}{\partial x_j \partial x_l}$$

Relation between the
Field Quantity Displacement and Elastic Parameters

If the displacement field can be observed, then materials parameters can be determined easily.

Numerical Treatment of Field Equations via Finite Element Method

BAM

Elastostatic case

$$0 = C_{ijkl} \frac{\partial^2 u_k}{\partial x_j \partial x_l} \quad \longrightarrow \quad \left(\sum_{i=1}^{21} C_i \mathbf{K}_i \right) \mathbf{u} = \mathbf{f} \quad \longrightarrow \quad \mathbf{u} = \left(\sum_{i=1}^{21} C_i \mathbf{K}_i \right)^{-1} \mathbf{f}$$

Due to the discretisation in the finite element method, it is no longer required to observed a complete continuous displacement field.

Integration of Finite Element Method in Least-Squares Adjustment

Introducing the vector of residual **v** for the displacement observation

$$\mathbf{u} + \mathbf{v} = \left(\sum_{i=1}^{21} C_i \mathbf{K}_i \right)^{-1} \mathbf{f}$$

$$\mathbf{L} = \begin{bmatrix} u_1 = 2.3\,\mathrm{mm} \\ u_2 = 2.8\,\mathrm{mm} \\ u_4 = 1.5\,\mathrm{mm} \\ u_7 = 4.3\,\mathrm{mm} \\ u_{14} = 8.8\,\mathrm{mm} \\ u_{59} = 1.5\,\mathrm{mm} \end{bmatrix} \qquad \mathbf{\Sigma_{LL}} = \begin{bmatrix} \sigma_u^2 & & & & & & 0 \\ & \sigma_u^2 & & & & & \\ & & \sigma_u^2 & & & & \\ & & & \sigma_u^2 & & & \\ & & & & \sigma_u^2 & & \\ & & & & & \sigma_u^2 & \\ 0 & & & & & & \sigma_u^2 \end{bmatrix}$$

Integration of Finite Element Method in Least-Squares Adjustment

Multivariate normal distribution

$$p = \frac{1}{\sqrt{(2\pi)^N \det(\mathbf{\Sigma_{LL}})}} \exp\left(-\frac{1}{2}\mathbf{v}^\mathsf{T}\mathbf{\Sigma}_\mathbf{LL}^{-1}\mathbf{v}\right) - 2\mathbf{k}^\mathsf{T}\mathbf{\Psi} \;\rightarrow \text{maximum}$$

looking for maximum or alternative

$$\Omega = \mathbf{v}^\mathsf{T}\mathbf{P}\mathbf{v} - 2\mathbf{k}^\mathsf{T}\mathbf{\Psi} \;\rightarrow \text{minimum} \qquad \mathbf{P} = \left(\frac{1}{\sigma_0^2}\mathbf{\Sigma_{LL}}\right)^{-1} = \mathbf{Q}_\mathbf{LL}^{-1}$$

with variance-covariance matrix of the observation.

Integration of Finite Element Method in Least-Squares Adjustment

"inverse" FEM

$$\Omega = \mathbf{v}^{\mathsf{T}}\mathbf{P}\mathbf{v} - 2\mathbf{k}^{\mathsf{T}}\mathbf{\Psi} \;\rightarrow \text{minimum}$$

$$\begin{bmatrix} \mathbf{B}\mathbf{Q}_{\mathrm{ll}}\mathbf{B}^{\mathsf{T}} & \mathbf{A} \\ \mathbf{A}^{\mathsf{T}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{k} \\ \mathbf{\Delta x} \end{bmatrix} = \begin{bmatrix} -\mathbf{w} \\ \mathbf{0} \end{bmatrix}$$

$$\mathbf{\Psi} = \left(\sum_{i=1}^{21} C_i \mathbf{K}_i \right) (\mathbf{u} + \mathbf{v}) - \mathbf{f} = \mathbf{0}$$

$$\mathbf{B} = \left. \frac{\partial \mathbf{\Psi}}{\partial \mathbf{v}} \right|_{\mathbf{v}^0,\, \mathbf{x}^0}$$

$$\mathbf{A} = \left. \frac{\partial \mathbf{\Psi}}{\partial \mathbf{x}} \right|_{\mathbf{v}^0,\, \mathbf{x}^0}$$

$$\mathbf{w} = -\mathbf{B}\mathbf{v}^0 + \mathbf{\Psi}^0$$

Integration of Finite Element Method in Least-Squares Adjustment

Gauss-Helmert Model and Gauss-Markov Model

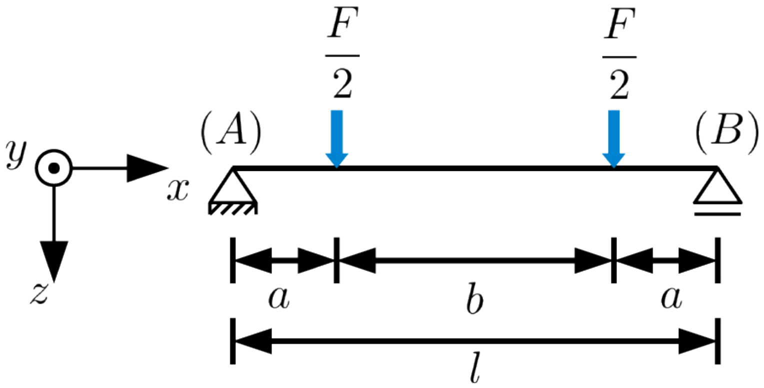
$$\Omega = \mathbf{v}^\mathsf{T}\mathbf{P}\mathbf{v} - 2\mathbf{k}^\mathsf{T}\boldsymbol{\Psi} \ \rightarrow \text{minimum}$$

$$\begin{bmatrix} \mathbf{B}\mathbf{Q}_{\mathbf{ll}}\mathbf{B}^\mathsf{T} & \mathbf{A} \\ \mathbf{A}^\mathsf{T} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{k} \\ \boldsymbol{\Delta}\mathbf{x} \end{bmatrix} = \begin{bmatrix} -\mathbf{w} \\ \mathbf{0} \end{bmatrix}$$

$$\Omega = \mathbf{v}^\mathsf{T}\mathbf{P}\mathbf{v} \ \rightarrow \text{minimum}$$

$$\left[\mathbf{A}^\mathsf{T}\mathbf{P}\mathbf{A}\right]\left[\boldsymbol{\Delta}\mathbf{x}\right] = \left[\mathbf{A}^\mathsf{T}\mathbf{P}\mathbf{l}\right]$$

Example: Euler-Bernoulli Beam



length	7.26 m
width	0.20 m
height	0.36 m
E-Modulus	70 GPa
Force	7460 N
a	2.42 m

726 finite elements are used

Example: Euler-Bernoulli Beam

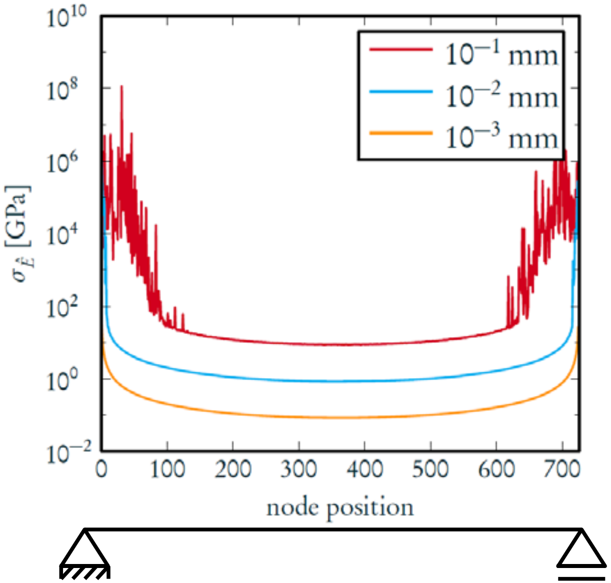
Using Euler-Bernoulli beam differential equation

$$\mathbf{u} = \left(\sum_{i=1}^{21} C_i \mathbf{K}_i \right)^{-1} \mathbf{f} \quad \Rightarrow \quad \mathbf{u} = (E\mathbf{K})^{-1} \mathbf{f} = \mathbf{A} \frac{1}{E} = \mathbf{A} X$$

each element is a fifth order hermite interpolation polynomial

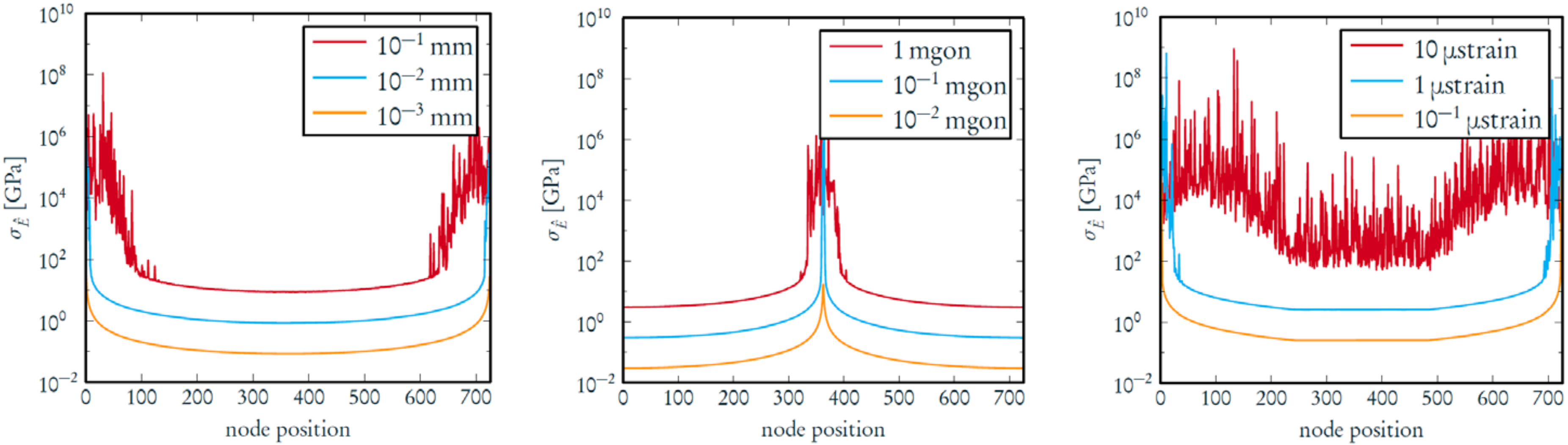
Example: Euler-Bernoulli Beam

Determination of optimal displacement sensor position

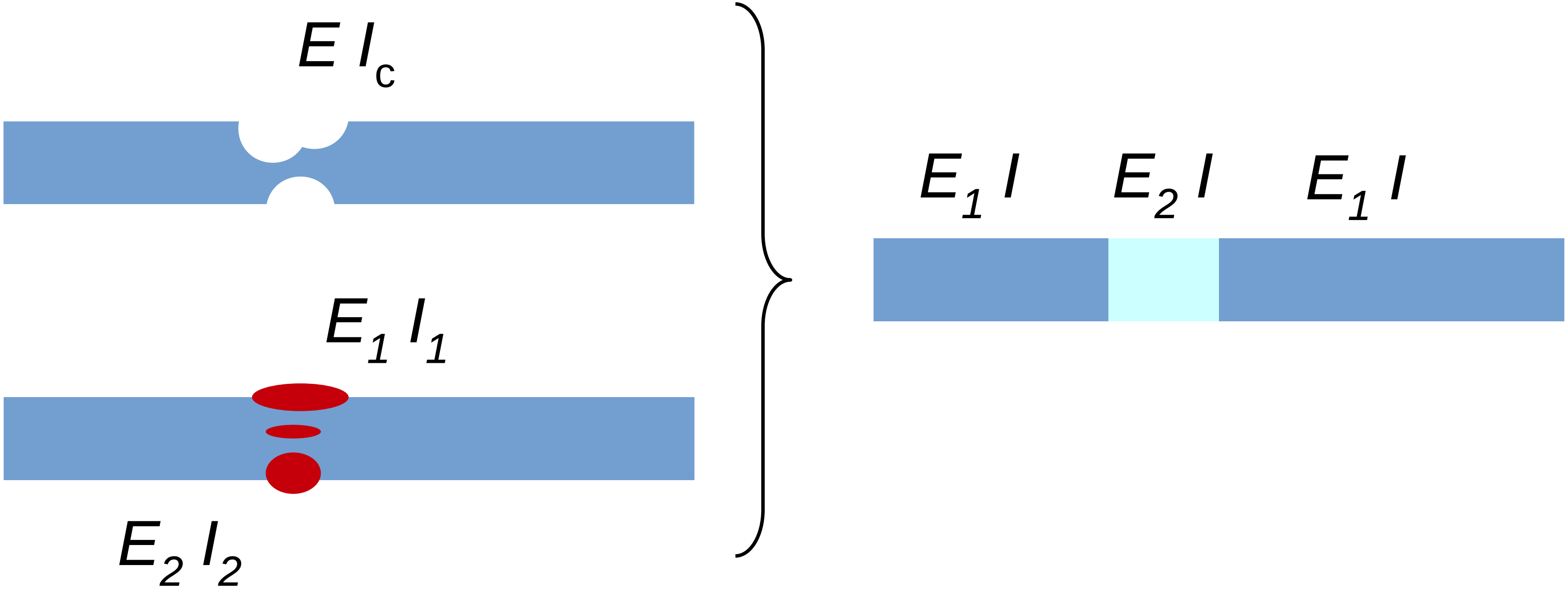


Example: Euler-Bernoulli Beam

Determination of optimal sensor position



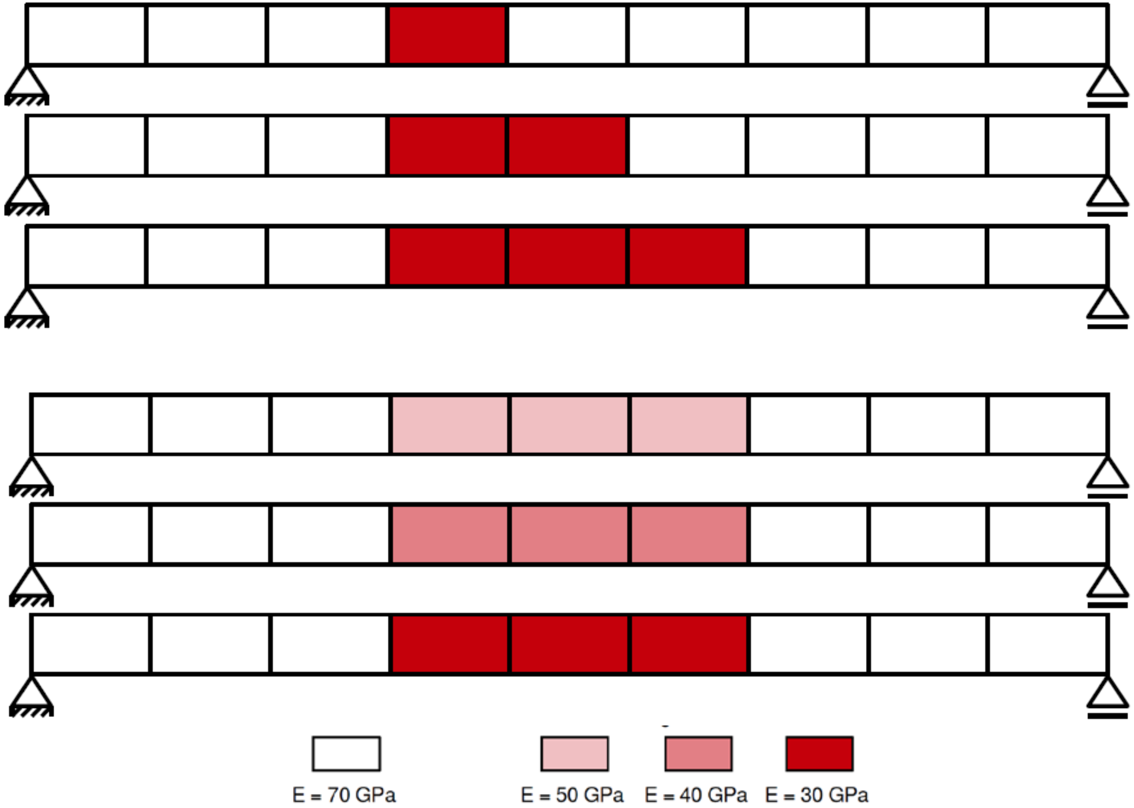
Example: Euler-Bernoulli Beam



Example: Euler-Bernoulli Beam

Scenario 1

Scenario 2



Example: Euler-Bernoulli Beam

How to detect and localise damage?

$$\begin{bmatrix} \mathbf{L} \\ \mathbf{X}_{\text{apriori}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{I} \end{bmatrix} \mathbf{X}$$

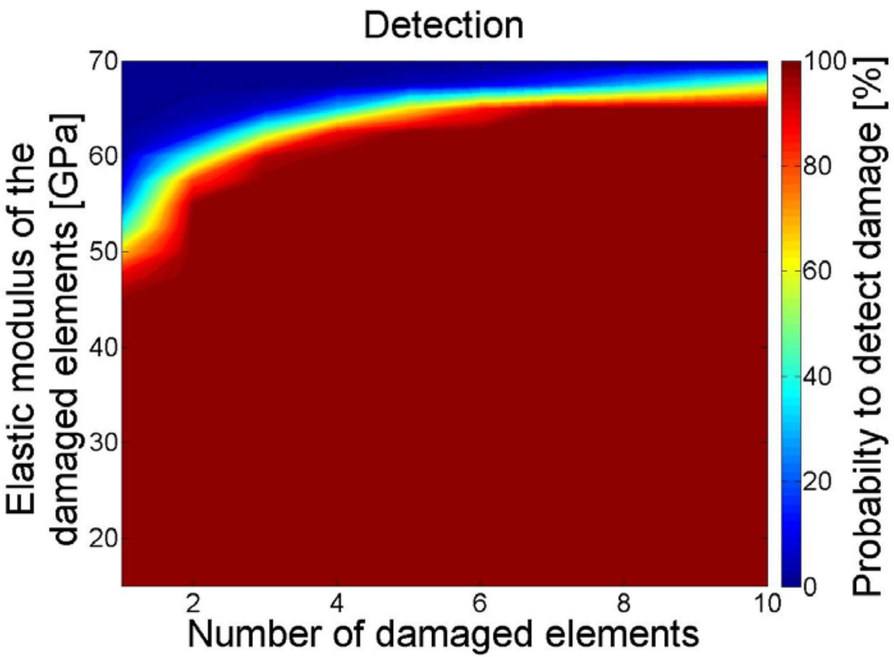
Does the empirical reference standard deviation after the adjustment coincides with the theoretical reference standard deviation?

$$H_0 : \mathrm{E}(s_0^2) = \sigma_0^2$$
$$H_\mathrm{A} : \mathrm{E}(s_0^2) \neq \sigma_0^2$$

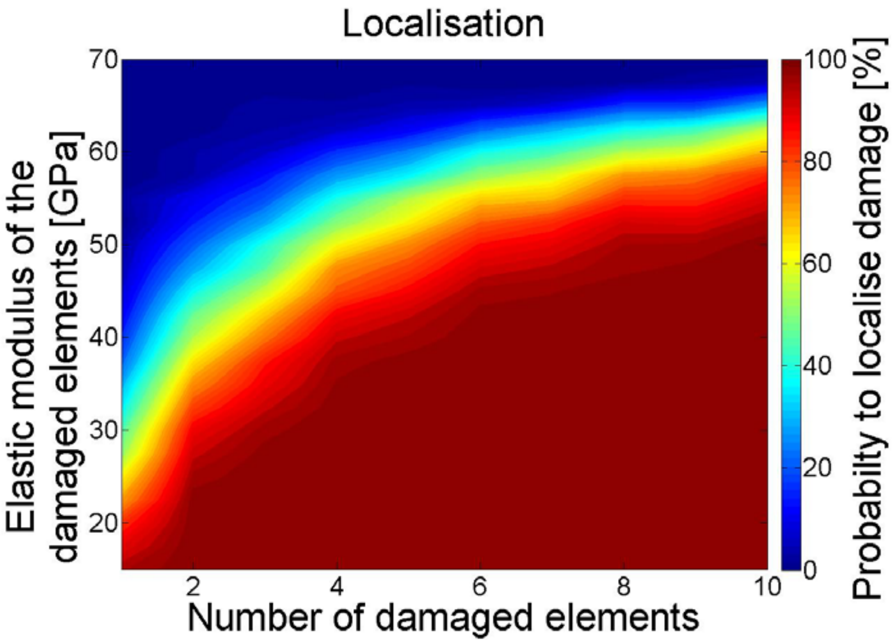
$$s_0 = \sqrt{\frac{\mathbf{v}^\mathsf{T}\mathbf{P}\mathbf{v}}{r}}$$

$$\chi_r^2 = r\,\frac{s_0^2}{\sigma_0^2}$$

Example: Euler-Bernoulli Beam



Example: Euler-Bernoulli Beam

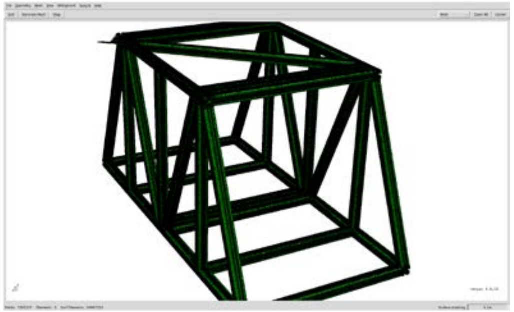


Outlook (Further aspects that are covered in my dissertation)

- The unavailability of measurements within a object
→ Two possible approaches

- Dealing with very complex structure by means of substitute model

- How to deal with measurements without models.



Discretisation Points: 7202237